

# AN ANISOTROPIC CREEP DAMAGE MODEL FOR MULTIAXIAL CYCLIC LOADING HISTORIES

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The purpose of this study is the development of an anisotropic creep damage theory within the continuum damage mechanics, applicable to creep-dominated cyclic loading histories. A damage distribution is expressed in rate form as a symmetric tensor of rank necessary to match physically measured damage. A theoretical model which expresses general anisotropic creep damage phenomena with power law cavity growth is proposed. The coupling of damage with a bounding surface cyclic viscoplasticity theory is also accomplished. Comparison with experimental results are made for weakly anisotropically damaging materials, type 304 stainless steel at 593°C. Good correlation of rupture time, secondary creep, and tertiary creep has been obtained for proportional and nonproportional, isothermal, constant isochronous nominal stress loading histories. A modification of the isochronous stress (the set of stress state which have a same rupture time) for compressive hydrostatic stress state has been offered.

**Key Words :** Creep, Continuum Damage, Anisotropic, Nonproportional

## NOMENCLATURE

$\underline{n}$  : Unit vector normal to the unit sphere  
 $w$  : Scalar damage distribution  
 $\underline{\sigma}$  : Cauchy stress tensor  
 $\underline{s}$  : Deviatoric stress tensor  
 $\underline{\phi}$  : Damage effect tensor  
 $\underline{\alpha}$  : Deviatoric backstress tensor  
 $\bar{\sigma}$  : Effective Cauchy stress  
 $\bar{\alpha}$  : Effective backstress  
 $\underline{\epsilon}$  : Strain rate tensor  
 $\underline{n} \otimes \underline{n}$  : Outer product between two vectors

## 1. INTRODUCTION

When external forces are applied to a structures, the resulting strains are comprised of elastic, plastic, and time-dependent components when thermal strains are subtracted. At high temperatures (approximately greater than 40% of the melting point), creep is an important phenomenon.

In early investigations of creep, researchers assumed that the strain rate was related only to the state of stress. In using this approach, however, it was very difficult to describe the behavior of damaged materials and the evolution of grain boundary damage which induces creep rupture. Hence, in a new analytical approach, damage parameters were used to describe the creep behavior of materials with significant levels of damage (e.g. tertiary creep stage).

By now, the theoretical application to isotropic creep damage has been well developed. This approach usually involves only a scalar damage parameter. However, in practical applications, damage phenomena are anisotropic and must be investigated experimentally. Physically, creep damage has a directional distribution which must be accounted

for.

In case of materials which obey a maximum tensile principal stress rupture criterion, e.g. copper, the lifetime of a specimen would be increased by rotating the applied principal stresses. The reason is that the damage growth rate on one set of planes would decrease after the rotation and damage growth commence on a fresh set of planes perpendicular to the new maximum principal stress direction. Theoretically, if a torque reversal is made close to the steady load lifetime for combined axial-torsional loading, and if no interaction takes place between the damage planes, then a lifetime of twice steady load lifetime should be achieved. Hence, the damage accumulation is clearly anisotropic in such a case.

In case of materials which obey a maximum effective stress rupture criterion, e.g. aluminum alloys, the damage rates should not depend upon the degree of proportionality of the loading. So the lifetime for a steady load test and reversed torsion test should be identical. Hence, the damage accumulation is essentially isotropic.

The purpose of this study is the development of an anisotropic creep damage theory and the establishment of appropriate evolution equations of damage rate and strain rate. In this study, various damage theories are reviewed. A theoretical model which expresses general anisotropic creep damage phenomena associated with power law governed cavity growth is proposed. The coupling of damage with cyclic viscoplastic deformation is also accomplished. Comparisons with experimental results are made for weakly anisotropically damaging materials.

## 2. SUGGESTION OF THEORETICAL MODEL

### 2.1 Creep Damage Growth Law

The approach taken by Murakami and Ohno (Murakami, 1983, Murakami and Ohno, 1980) results in a second rank tensor damage distribution regardless of the nonproportionality of the applied loading history. Some

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experiments (Hayhurst and Leckie, 1985, Oyata, 1982, etc.), however, revealed that the creep damage distribution for some materials is highly dependent on principal stress orientation. Leckie and Onat (1980, 1983) suggest stronger conditions, based on damaged material symmetry arguments, which require that a vectorial distribution of damage be even with respect to  $\underline{n}$ , i.e.

$$\omega(\underline{n}) = \omega(-\underline{n}) \quad (1)$$

where  $\underline{n}$  is a unit vector normal to the unit sphere and  $\omega(\underline{n})$  is the distribution of damage.

Following the development of Leckie and Onat (1980, 1983), we define the damage distribution  $\omega(\underline{n})$  in terms of a set  $\underline{\Gamma}$  of even rank, irreducible tensors obtained from the distribution of damage on the unit sphere. Effective stress  $\underline{S}$  may be expressed as a second rank tensor function of Cauchy stress  $\underline{\sigma}$  and a fourth rank operator  $\underline{M}(\underline{\Gamma})$ , i.e.

$$\underline{S} = \underline{S}[\underline{M}(\underline{\Gamma}), \underline{\sigma}] \quad (2)$$

In this work, we make the simplifying assumption that the principal axes of the Cauchy stress and effective stress coincide, which may be true for proportional loading even up to large cavity volume fractions. However, for nonproportional loading such an assumption suggests a limitation to relatively small cavity volume fractions to ensure that the rotation of the effective stress with respect to the Cauchy stress is suitably small. With this assumption we may express  $\underline{M}$  in the principal stress coordinate frame as

$$M_{ijkl}(\underline{\Gamma}) = \phi_{ij}(\underline{\Gamma}) \delta_{ik} \delta_{jl} \quad (\text{no sum on } i \text{ and } j) \quad (3)$$

$$\underline{\phi} = \mathcal{Q}^{(j)} \underline{n}^{(j)} \otimes \underline{n}^{(j)} \quad (4)$$

where  $\underline{\phi}$  is the damage effect tensor with principal components  $\mathcal{Q}^{(j)}$  and eigenvectors  $\underline{n}^{(j)}$  collinear with those of  $\underline{\sigma}$ .  $\underline{\phi}$  is viewed as an approximation of the intensification effect of grain boundary damage on the current principal stresses. We define the growth rate of  $\omega$  as a function of  $\underline{\phi}$  and  $\underline{\sigma}$  in the following simple way:

$$\dot{\omega}(\underline{n}) = \xi(\sigma^*) [\eta \chi^{(1)} \{\mathcal{Q}^{(1)}\}^{\lambda(\sigma^*)} + (1-\eta) \sum \chi^{(j)} \{\mathcal{Q}^{(j)}\}^{\lambda(\sigma^*)}] \times \{ \underline{n} \cdot \underline{n}^{(j)} \otimes \underline{n}^{(j)} \cdot \underline{n} \} \{ \underline{n} \cdot \underline{n}^{(j)} \}^{2P} ] = \dot{\omega}(\underline{n})_{\text{isotropic}} + \dot{\omega}(\underline{n})_{\text{anisotropic}} \quad (5)$$

anisotropic damage rate tensor  $\underline{\dot{\Gamma}}$  of rank  $2(P+1)$ , i.e.

$$\underline{\dot{\Gamma}} = \xi(\sigma^*) [\eta \chi^{(1)} \{\mathcal{Q}^{(1)}\}^{\lambda(\sigma^*)} \otimes_{i=1}^{p+1} \underline{I} + (1-\eta) \sum_{j=1}^3 \chi^{(j)} \{\mathcal{Q}^{(j)}\}^{\lambda(\sigma^*)} \otimes_{i=1}^{2(p+1)} \{ \underline{n}^{(j)} \} ] \quad (6)$$

where  $\xi$  and  $\lambda$  are functions of the isochronous stress  $\sigma^*$ ,  $\underline{n}^{(j)}$  is the unit vector in the  $j$ th principal stress direction,  $P$  is an integer of the order of the anisotropic damage distribution, outer product operator was multiplied by  $(p+1)$  and  $2(p+1)$  times respectively, and  $\eta$  is the fraction of damage rate in the  $\underline{n}^{(1)}$  direction which is isotropic. Factor  $\chi^{(j)}$  excludes contribution of compressive principal stresses to the damage rate, i.e.  $\chi^{(j)} = 0$ .

$$\chi^{(j)} = \langle \underline{n}^{(j)} \cdot \frac{\underline{\sigma}}{|\underline{\sigma}_j|} \cdot \underline{n}^{(j)} \rangle \quad (7)$$

where  $\sigma_j$  are the ordered principal stresses with  $\sigma_1 > \sigma_2 > \sigma_3$ , and the  $\langle \rangle$  is Macauley bracket ( $\langle Y \rangle = Y$  if  $Y > 0$ ,  $\langle Y \rangle = 0$  otherwise).

Among several proposed  $\mathcal{Q}^{(j)}$ , one possibility is as follow:

$$\mathcal{Q}^{(j)} = \left[ \frac{1}{(1-\omega(\underline{n}^{(j)}))^2} + \frac{2}{(1-\eta-\omega(\underline{n}^{(j)}))^2} \right]^{1/2} \quad (8)$$

Essentially, this equation assumes that stress intensification effect associated with each of the damage values  $\omega(\underline{n}^{(j)})$  is represented by a symmetric term but dependent on  $\eta$ . The scalar multiplier  $\mathcal{Q}^{(j)}$  directly affects evolution of the  $\omega$  distribution, as seen in Eq.(5). It should be noted that for isotropic hardening, the damage rate equation assumes the classical Kachanov-Rabotnov form. It is necessary to introduce a specific definition for the isochronous stress discussed by Trampczynski and Hayhurst (1980) and Lemaitre and Chaboche (1982, 1975). Our suggested isochronous stress is

$$\sigma^* = \left(\frac{3}{2}\right) S_1 \left[\frac{2}{3} \frac{\sigma_e}{S_1}\right]^a \exp[b(1+f(J_1)\langle -J_1/|J_1| \rangle) \left(\frac{J_1}{S_s} - 1\right)] \quad (9)$$

$$S_1 = \sigma_1 - (1/3) \sigma_{kk}, \quad \sigma_e = [(3/2) \underline{s} : \underline{s}]^{1/2} \quad (10), \quad (11)$$

$$\underline{s} = \underline{\sigma} - (1/3) \sigma_{kk} \underline{I}, \quad S_s = [\sigma_1^2 + \sigma_2^2 + \sigma_3^2]^{1/2}, \quad J_1 = \sigma_{kk} \quad (12, 13, 14)$$

It should be noted this form of the isochronous stress is a slightly altered form of that proposed by Huddleston (1984). This form of the isochronous stress has been rather thoroughly supported by a variety of biaxial creep experiments on tubular specimens of type 304 stainless steel at 593°C at ORNL (Oak Ridge National Laboratory), including axial tension, equi-biaxial tension, internal pressure, torsion, axial tension and torsion, and axial compression and torsion.

Obviously, the integrated damage distribution  $\omega(\underline{n})$  will depend on whether the loading history is proportional or nonproportional. Rotation of the principal stress eigenvectors will in general result in multiple peaks in the damage distribution with respect to a fixed material coordinate system.

Another key element of the damage formulation is the rupture criterion. If a stress level-dependent rupture criteria is adopted, than the damage at rupture is not constant and the time fraction at any given damage level depends on the isochronous stress for proportional loading. A stress level-dependent rupture criterion is more difficult to implement since experimental investigation of the damage distribution at different stress levels is quite involved. The logical approximation to the physically more precise stress level-dependent rupture criterion is the first approach, i.e. the assumption of a constant damage at failure. Therefore, from a practical viewpoint, the use of a constant damage at rupture is likely to be sufficient, especially in view of the inherent scatter in creep rupture tests. Hence, the rupture criterion  $\omega_{\max} = \max \omega(\underline{n}) = 1$  is selected in this work.

### 3. RATE-DEPENDENT BOUDING-SURFACE FORMULATION ; TYPE 304 STAINLESS STEEL AT 593°C

#### 3.1 Theoretical Equation

To describe rate-dependent, high-temperature creep-

plasticity response for histories which involve both creep and cyclic plasticity, it is necessary to introduce a theory capable of modeling a wide range of behavior. For multiaxial cyclic plasticity, it has previously been demonstrated that a bounding surface approach (Dafalias, 1981, McDeowell, 1985, etc.) provides very good correlation of nonproportional deformation behavior. Since such behavior is of concern to the current investigation, a new strain-hardening model based on a rate-dependent bounding surface with a Mroz translation rule for backstress is adopted. Key features of this theory include isotropic hardening reflected through growth of the bounding surface rather than a scalar parameter in the flow rule, and rate-dependence of the backstress evolution even at high strain rates. In McDowell study (1985), the backstress was defined as the kinematic hardening parameter in case of general nonproportional loading. These features contrast with conventional unified creep plasticity models (Chan, 1984, Hart, 1976, etc.). Rate-dependence is reflected primarily through bounding surface dependence on overstress, strain-hardening is reflected through growth of the bounding surface, and smooth yielding response is obtained through use of the Mroz distance vector in the backstress hardening rate coefficient. Briefly, the damage-coupled bounding surface model can be stated in multiaxial form as

$$\dot{\epsilon}^n = (3/2) K < \bar{\sigma} D - K_o >^n \exp(Z < \bar{\sigma} D - K_o >^{n+1}) \frac{(s - \underline{\alpha}) \sqrt{\sigma}}{\sigma} \quad (15)$$

$$\bar{\sigma} = [(3/2) (s - \underline{\alpha}) : (s - \underline{\alpha})]^{1/2}, \quad (16), \quad (17)$$

$$\bar{\alpha} = [(3/2) \underline{\alpha} : \underline{\alpha}]^{1/2}$$

Here  $s$  and backstress  $\underline{\alpha}$  are deviatoric stress, and we have defined  $K_o = \text{constant}$ . The inelastic strain  $\epsilon^n$  includes both conventional creep and plastic strain as in other unified theories. The effective overstress and backstress are denoted as  $\bar{\sigma}$  and  $\bar{\alpha}$ , respectively. The exponential term was proposed by Nouailhas (1987) for description of high strain rate events. Note that the effect of damage is reflected by a multiplicative factor  $D$ . Hence, we define the factor  $D$  as

$$D = 1 + C + \Psi^m \quad (18)$$

where  $\Psi$  is defined as the average value of  $\omega(\eta)$  along the unit sphere area and  $C$  and  $m$  are material constants.

The competition between hardening and static thermal recovery terms in the backstress rate equation is introduced in this bounding surface formulation in the following way:

$$\dot{\bar{\alpha}} D = H(\bar{\alpha} D, \delta) \|\dot{\epsilon}\| \nu - R(\bar{\alpha} D) \bar{\sigma} D \quad (19)$$

$$H(\bar{\sigma} D, \delta) = \beta_o + \beta_1 \exp(-\beta_4 < 1 - \beta_3 \left( \frac{\delta}{R^*} \right) >^{\beta_5}) + \beta_2 \exp(-\beta_6 \bar{\alpha}^* D) \quad (20)$$

$$\delta = (3/2)^{1/2} \|(2/3)^{1/2} R^* \underline{N} - \underline{s}\|, \quad \underline{N} = \frac{(s - \underline{\alpha}) \sqrt{\sigma}}{\sigma} \quad (21), \quad (22)$$

$$\bar{\alpha}^* = (R^* - \delta - \bar{\sigma}) \quad (23)$$

$$R(\bar{\alpha} D) = \beta_7 \exp(-\beta_8 \bar{\alpha} D) (\bar{\alpha} D)^{\beta_9} \quad (24)$$

The radius of the bounding surface,  $R^*$ , evolves with accumulated plastic strain (creep hardening) and responds through the effective overstress to changes in inelastic strain rate, i.e.

$$R^* = \beta_{10} \rho [1 + \beta_{12} \bar{\sigma}^{\beta_{13}}] \quad (25)$$

$$\dot{\rho} = \beta_{11} (\rho_{mf} - \rho) (2/3)^{1/2} \|\dot{\epsilon}\|^n \quad (26)$$

The directional index,  $\nu$ , for the backstress hardening rate is a rate-dependent Mroz form (Mroz, 1976). In this formula-

tion  $K$ ,  $K_o$ ,  $Z$ ,  $n$ ,  $c$ ,  $m$ ,  $\beta_1, \dots, \beta_{13}$ , and  $\rho_{mf}$  are isothermal material constants. Non-isothermal generalization can be achieved primarily by invoking temperature dependence of the backstress recovery term (Murakami and Ohno, 1980) and some of the constants, although this is not necessary in the current isothermal work.

### 3.2 Determination of Material Constants

All material constants were determined from uniaxial creep tests found in the literature and by examination of ruptured uniaxial specimens. Constants  $a$  and  $b$  in isochronous stress level were given by Huddleston (1984) for type 304 stainless steel at 593°C as  $a = 1.086$ ,  $b = 0.289$ .

Interrupted creep tests are generally necessary to assess exponent  $\lambda$  in Eqs. (5, 6) at a given isochronous stress level;  $\lambda$  can also be determined in an approximate way by periodically unloading from the creep curve (Chaboche, 1982, Lemaître and Chaboche, 1975) or by matching the integrated damage-coupled creep Eqs. (15~26) with observed onset of tertiary response assuming  $m = 1$ . The value  $m = 1$  arises from the Murakami study (1983). Stress exponent  $k$  is easily identified as the slope of the  $\log(t_R)$  vs.  $\log(\sigma)$  curve obtained from uniaxial tests. isotropic damage fraction  $\eta$  is identified as the ratio of the transverse damage to the longitudinal damage in a uniaxial tests, and is identified by quantitative metallographic techniques. As the isochronous stress level of this study, cavity growth is governed by matrix power law creep. Hence, we define

$$\xi(\sigma^*) = B[\sigma^*]^k \quad (27)$$

Once  $k$  is known, coefficient  $B$  can be found at the isochronous stress level associated with  $\lambda$  by integrating and matching rupture times from uniaxial tests with the assumed rupture criterion  $\omega_{max} = 1$ . For an isotropic stress of 176.1 MPa at 593°C, the constants are  $b = 2.71 \times 10 \text{sec}^{-1}$  and  $\lambda = 4.8$ . Units of stress and damage rate are MPa and  $\text{sec}^{-1}$ , respectively. Constants independent of isochronous stress include  $k = 8.5551$  and  $\eta = 0.61$ . Quantitative evaluation of the grain boundary damage distribution for these biaxial experiments revealed that the damage distribution is suitably described by a second rank tensor (McDowell and Ho, 1986) which sets  $P = 0$ ;  $\eta$  was also estimated by quantifying grain boundary metallographs. Exponent  $\lambda$  was estimated by matching the tertiary response of uniaxial tests with the integrated damage-coupled creep equations.

Strain rate sensitivity data of Steichen (1976) for annealed type 304 stainless steel was used to determine constants  $\beta_{10}$ ,  $\beta_{12}$ , and  $\beta_{13}$ . Constant  $K_o$  selected as representative of initial yielding of the annealed material; Constants  $K$  and  $n$  were selected on the basis of the relatively high ratio between backstress magnitude and deviatoric stress magnitude observed in nonproportional biaxial creep experiments (Oyata, Delobelle and Meret, 1982) on a somewhat similar material. Constants  $\beta_o, \dots, \beta_8$ ,  $\rho_{mf}$ , and  $\beta_{11}$  were determined from data obtained by Corum (1974) on the same ORNL reference heat of annealed material at 593°C. Constants  $\beta_7$ ,  $\beta_8$ , and  $\beta_9$  were determined after the hardening terms were established by fitting the secondary creep rate versus stress relationship established from numerous uniaxial tests at ORNL at 593°C. The damage coefficient  $C$  was obtained with the assumption  $m = 1$  by fitting the tertiary portion of the creep response. In summary, the material constants at 593°C for the damage-coupled creep-plasticity model are:

$$\begin{aligned}
 K &= 5 \times 10^{-48}, K_0 = 13.8, n = 30, \beta_0 = 1104, \beta_1 = 6.9 \times 10^6 \\
 \beta_2 &= 1044, \beta_3 = 1.18, \beta_4 = 23.16, \beta_5 = 1.25, \beta_6 = 0.0196 \\
 \beta_7 &= 1.55 \times 10^{-19}, \beta_8 = -0.0207, \beta_9 = 5.088, \beta_{10} = 0.0037 \\
 \beta_{11} &= 7, \beta_{12} = 0.225, \beta_{13} = 1.91, \rho_{mf} = 517, \rho_0 = 145 \\
 C &= 0.32, m = 1, Z = 0,
 \end{aligned}$$

where the units of stress are in MPa and time in sec.

3.3 Discussion of Results

Model predictions and experimental data (McDowell and Ho, 1986) for different biaxial creep experiments are shown in Figs. 1, 2, 3, 4. In these figures, inelastic axial and shear strain components which were defined in Eqs. 15~26 are plotted versus time in addition to the axial and shear stress history which are nominal values based on initial dimensions. The shear stress was applied at the bottom of each specimen in either a clockwise or counter-clockwise sense viewing down the specimen longitudinal axis. It could be noted that the isochronous stress for all specimens is constant at  $\sigma^* = 176.15$  MPa. Hence, differences in rupture time should be attributable to nonproportionality of loading, neglecting the possibility of reversal of plastic strain. Specimens GT-1 and GT-2 were subjected to identical loading histories apart from the sign of shear stress. The loading history of specimen GT-3 involved a reversal in sign of the shear stress at 456 hours and the loading history of specimen GT-6 involved a repeated reversal(cyclic loading).

It is observed that a rupture time of 998 hours is predicted for Specimen GT-1 and GT-2; the actual values are 892 hours and 1173 hours, respectively. The inelastic behavior and rupture time is well predicted for proportionally loaded Specimen GT-1 and GT-2. It is noted that there is the excellent agreement of the onset of tertiary creep and magnitude of creep strain at rupture between theory and experiment.

Also note that the slope of the predicted creep rate is not infinite at rupture, which is confirmed by the experiments. The actual grain boundary microcrack damage at rupture is significantly less than for specimen GT-1 and GT-2, with a maximum damage of  $\max \omega(n) = \omega(n^{(1)}) = 0.28$  in the maximum tensile principal stress direction. This result provides confirmation that the rupture event is a process of unstable linkage and propagation of grain boundary cracks after a critical level of grain boundary damage is reached. It is important to note that the rupture criterion used in this calculation,  $\max \omega(n) = 1$ , may be replaced by any other, more realistic criterion such as  $\max \omega(n) = 0.25$ , provided that the coupling constants  $C$  and  $m$  in the creep strain rate equation are determined in conjunction with this criterion. It is concluded that the even rank damage tensor approach of this study offers an improvement on prior isotropic damage models by assigning magnitude and direction of creep damage for proportional loading.

The purpose of experiment GT-3 was to evaluate the capability of the tensor damage model to predict direction

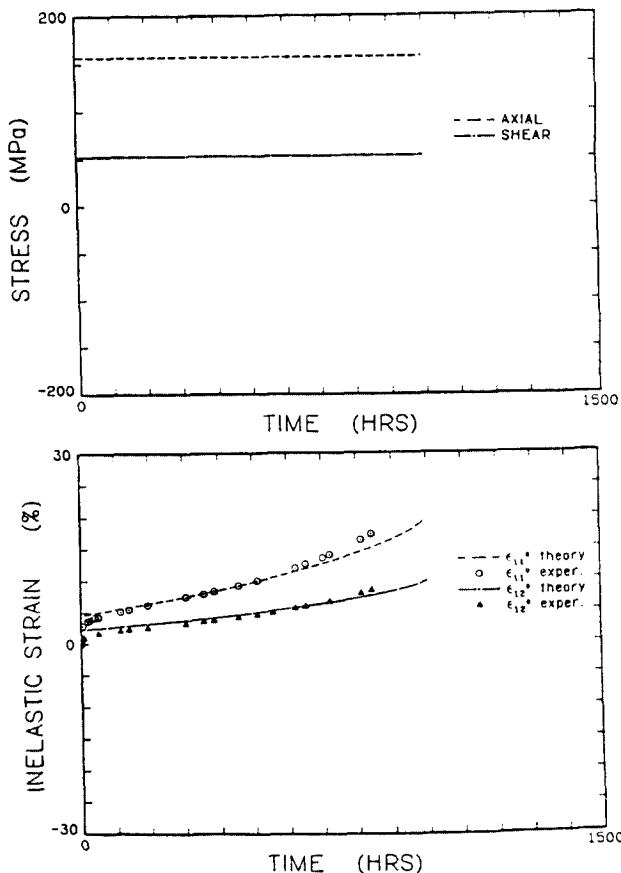


Fig. 1 Type 304 stainless steel at 593°C : applied biaxial nominal stress history (top) and predicted versus experimental inelastic strains (bottom) for GT-1. Actual and predicted rupture times are 892 hr and 998 hr, respectively, Theoretical strains are plotted by using, Eqs. (15~26)

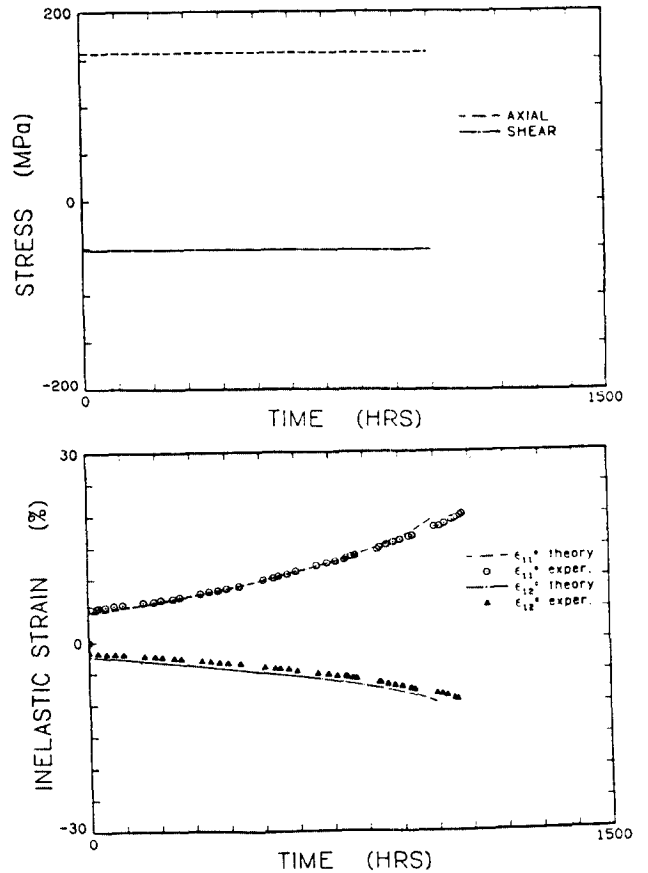


Fig. 2 Type 304 stainless steel at 593°C : applied biaxial nominal stress history (top) and predicted versus experimental inelastic strains (bottom) for GT-2. Actual and predicted rupture times are 1173 hr and 998 hr, respectively, Theoretical strains are plotted by using Eqs.(15~26).

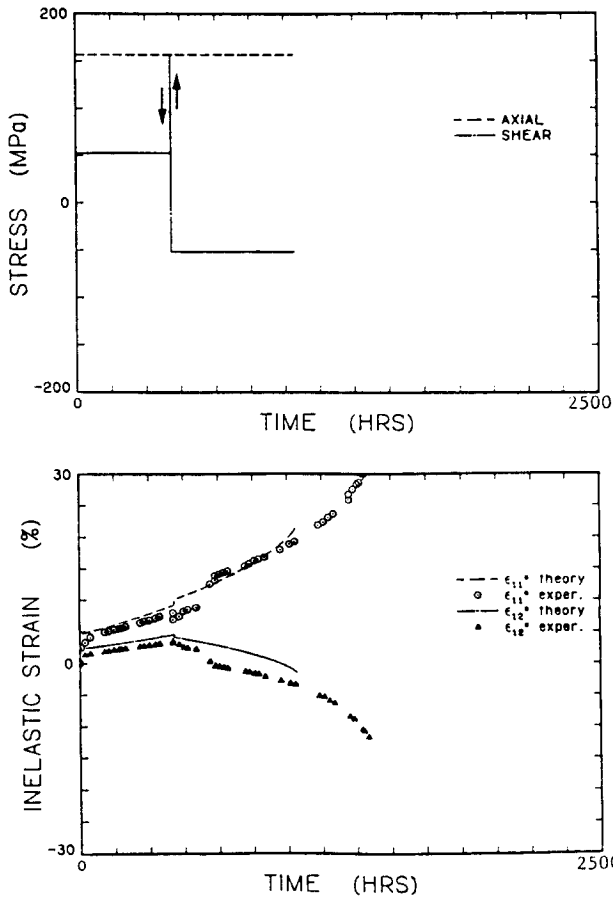


Fig. 3 Type 304 stainless steel at 593°C : applied biaxial nominal stress history (top) and predicted versus experimental inelastic strains (bottom) for GT-3. Actual and predicted rupture times are 1398 hr and 1043 hr, respectively, theoretical strains are plotted by using Eqs.(15~26).

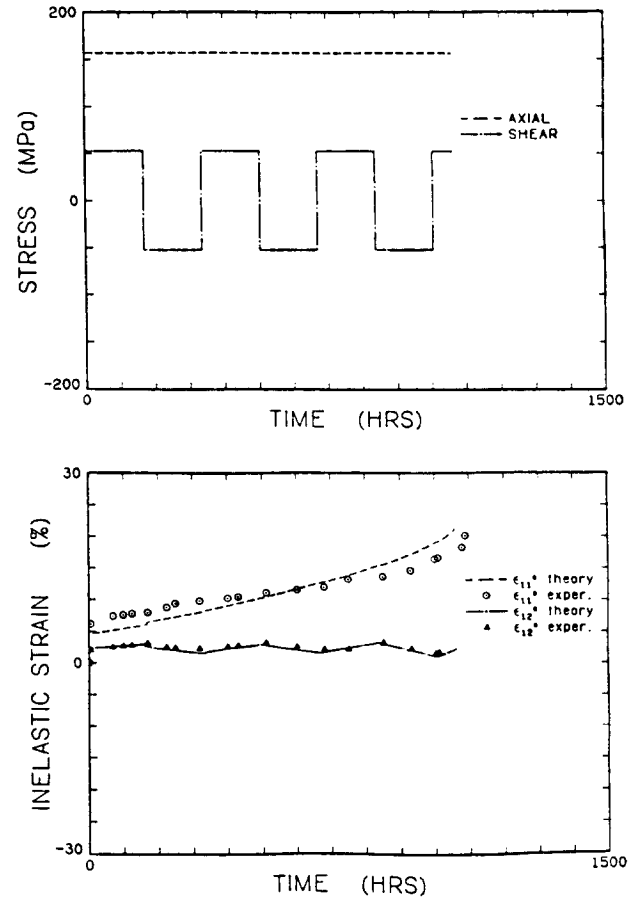


Fig. 4 Type 304 stainless steel at 593°C : applied biaxial nominal stress history (top) and predicted versus experimental inelastic strains (bottom) for GT-6. Actual and predicted rupture times are 1088 hr and 1060 hr, respectively, theoretical strains are plotted by using Eqs.(15~26).

and magnitude of accumulated creep damage in addition to rupture time for nonproportional loading histories. As shown in Fig.3, specimen GT-3 was subjected first to a counter-clockwise shear stress and an axial stress at the same isochronous stress level as specimens GT-1 and GT-2; after 456 hours, however, the shear stress was reversed while maintaining the same isochronous stress level. A maximum principal stress rotation of  $34^\circ$  resulted from this shear stress reversal. It is noted first that the predicted rupture time of 1043 hours is significantly lower than the observed 1398 hours. Also, the measured orientation of maximum damage is  $13^\circ$  from the predicted orientations. This result indicates that the actual rotation of the damage tensor was not as great as the predicted by the theory. Also, note that the magnitude of damage at rupture is less than in proportionally loaded specimen GT-1 and GT-2.

It is observed that a rupture time 1060 hours is predicted for Specimen GT-6; the actual value is 1088 hours. The correlation obtained for GT-6, a somewhat complex creep-dominated cyclic loading history, is quite good.

#### 4. CONCLUSIONS

The present study has demonstrated that anisotropic creep

damage can be successfully treated within the framework of continuum damage mechanics, even for creep-dominated cyclic loading histories typical of nuclear components. A damage distribution with even symmetry has been introduced on the unit sphere; this distribution evolves in rate form as a symmetric tensor of rank necessary to match physically measured damage distributions. The approach is motivated by the treatment of even rank tensor distributions forwarded by Leckie and Onat (1980, 1983), and contains as a subset the specific tensorial definitions of damage adopted in the anisotropic theories of Chaboche (1979, 1981) and Murakami and Ohno (1983, 1980).

A general form of coupling with damage has been suggested for an internal variable inelasticity framework and specific forms have been developed for type 304 stainless steel at 593°C with the assumption of small cavity volume fractions. A novel bounding surface cyclic viscoplasticity theory has been offered for multiaxial creep-plasticity deformations as one of these specific forms. Good correlation of rupture time, secondary creep, and tertiary creep has been obtained for proportional and nonproportional, isothermal, constant isochronous nominal stress loading histories. A modification of the isochronous stress compressive hydrostatic stress states has been offered.

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